Nagaoka ferromagnetism in large-spin fermionic and bosonic systems

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We study the magnetic properties of itinerant quantum magnetic particles, described by a generalized Hubbard model with large spin ($S>1/2$), which may be realized in optical lattices of laser-cooled atom systems. In fermion systems (half-integer spins), an extended form of Nagaoka ferromagnetism may be realized. However, as novel aspects of the large-spin cases, we found that the condition on the lattice connectivity is more stringent than in the case of $S=1/2$ particles and that the system shows a peculiar degenerate structure of the ground state in which the ferromagnetic state is included. In contrast, it turns out that the ground state of itinerant bosonic systems (integer spins) has a degenerate structure similar to that of fermion system with $S>1/2$ regardless of the shape, connectivity, or filling of the lattice, and that the state with the maximum total spin is always one of the ground states. Because the system consists of $2S+1$ types of particles and we study a SU($2S+1$) invariant model, the degeneracy of the ground state is given by the multiplets of the fully symmetric Young tableau of SU($2S+1$) if the state with maximum total spin belongs to the ground state.

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I. INTRODUCTION

The origin of magnetism is attributed to the quantum-mechanical interaction of particles, which carry spin. In the so-called localized spin systems, the Pauli principle plays an essential role and the magnetic interaction is expressed by the Heisenberg interaction,[1] the exchange integral between atoms being the dominant term. Not only the two-spin interaction but also multispin interactions may contribute to give rise to exotic magnetic states. In particular, various effects of the multispin interaction in ^{3}He have been reported.[2,3] For itinerant electron systems, the origin of the magnetic order, and of ferromagnetic order, in particular, has been studied extensively. Itinerant electron systems are often described by tight-binding models such as the Hubbard model.[4] Nagaoka pointed out that the ground state of the Hubbard model may be a ferromagnetic state (Nagaoka-ferromagnetic state) if the number of electrons is reduced by one from half-filling, where half-filling means that the number of particles is the same as the number of the lattice sites.[5] This interesting ferromagnetic state has been studied extensively and its mathematical structure is well understood.[6,7] Various properties of this state have been explored, see, for example, Refs. 8–10. The Nagaoka-ferromagnetic state is established if the so-called "connectivity condition" (to be explained later) on the lattice is satisfied. In the Nagaoka-ferromagnetic state, the energy of the system is minimized when the total spin of the system takes the maximum value. On the other hand, the ground state of the half-filled system is a singlet state, that is, its total spin is zero. Elsewhere, we have studied how the magnetic state changes between these two states when an electron is removed from the system and demonstrated an adiabatic change between these states.[11]

Magnetism has primarily been studied in electron systems for which the spin $S=1/2$. However, recently, developments in the field of laser-cooled atomic systems have opened new possibilities to realize artificial tight-binding quantum system such as the Hubbard model.[12,13] In contrast to the electron systems, in optical lattices the spin of trapped atoms is not necessarily $S=1/2$ but can take larger half-integer or integer values. In the latter case, the system contains bosons, not fermions. It is, therefore, of interest to study itinerant magnetism of systems with $S>1/2$. The present paper presents a study of such systems.

II. MODEL

We consider a tight-binding model, which consists of the hopping term and a repulsive on-site interaction

$$\mathcal{H} = -t\sum_{\langle ij\rangle,M}(c_{i,M}^{\dagger}c_{j,M} + c_{j,M}^{\dagger}c_{i,M}) + \sum_{i=1}^{N} U(n_{i,M}), \qquad (1)$$

where $c_{i,M}^{\dagger}$ and $c_{i,M}$ are the annihilation and creation operators of a particle (fermion or boson depending on its spin) of the magnetization M at a site i, respectively, $n_{i,M}$ is the number operator

$$n_{i,M} = c_{i,M}^{\dagger}c_{i,M}, \qquad (2)$$

for the particle of spin S, $M=-S,-S+1,\ldots,S$, and $U(n_{i,M})$ represents the on-site repulsive interaction. For the system with $S=1/2$, we take for $U(n_{i,M})$ the standard form

$$U(n_{i,M}) = U_0 n_{i,-1/2}n_{i,1/2}, \qquad (3)$$

and for larger $S>1/2$ we extend it as

$$U(n_{i,M}) = \frac{1}{2}U_0 N_i(N_i - 1), \tag{4}$$

where $N_i=\Sigma_M n_{i,M}$ and U_0 is assumed to be positive. While the repulsion is usually attributed to the Coulomb interaction in the case of $S=1/2$, we adopt this form for the cases of $S>1/2$ by assuming that multiple atoms on the same site would cause the energy to increase. The detailed form of the interaction $U(n_{i,M})$ does not affect the conclusion of the present study as far as the energy increases significantly when a site is occupied by more than one particle.

It should be noted that the present model has SU($2S+1$) symmetry.[14] Although this symmetry property depends on the form of $U(n_{i,M})$, the symmetry of the hopping term plays major role in the present study and our result will not change as long as there is a strong repulsive interaction on each site.

In order to study magnetic properties of the system, we introduce spin operators for the magnetization $\boldsymbol{S}=(S_i^x,S_i^y,S_i^z)$ of the particles, where

$$S_i^z = \sum_{M=-S}^{S} M n_{i,M}. \tag{5}$$

Hereafter, we use the operators $S_i^+=S_i^x+iS_i^y$ and $S_i^-=S_i^x-iS_i^y$, which are expressed in terms of $c_{i,M}^\dagger$ and $c_{i,M}$

$$S_i^+ = \sum_M \sqrt{(S-M)(S+M+1)}c_{i,M+1}^\dagger c_{i,M} \tag{6}$$

and

$$S_i^- = \sum_M \sqrt{(S+M)(S-M+1)}c_{i,M-1}^\dagger c_{i,M}, \tag{7}$$

where S is the total spin of each particle, and M is the z component of the magnetization. To discuss the magnetic properties of the states, we adopt the usual notation $|S,M\rangle$, where S is the total spin S and M is the magnetization. The action of the operators in Eqs. (6) and (7) on the state $|S,M\rangle$ is given by the relations

$$S_i^+|S,M\rangle = \sqrt{(S-M)(S+M+1)}|S,M+1\rangle \tag{8}$$

and

$$S_i^-|S,M\rangle = \sqrt{(S+M)(S-M+1)}|S,M-1\rangle. \tag{9}$$

In order to explicitly compute matrix elements of the operators, it is convenient to introduce orthonormal basis states. In the case of fermion systems, we adopt the form

$$|\Psi\rangle = c_{i,M}^\dagger \cdots c_{j,M'}^\dagger|0\rangle, \tag{10}$$

where $i\geq j$, and $M>M'$ if $i=j$. With this definition, the operations in Eqs. (6) and (7) do not change the order of creation operators in Eq. (10). In the case of bosons, more than two particles with the same M can occupy the same site and the normalized basis states take the form

$$|\Psi\rangle = \frac{(c_{i,M}^\dagger)^{n_{i,M}}\cdots(c_{j,M'}^\dagger)^{n_{j,M'}}|0\rangle}{\sqrt{n_{i,M}!}\cdots\sqrt{n_{j,M'}!}} \equiv |n_{i,M}\rangle\cdots|n_{j,M'}\rangle. \tag{11}$$

For boson systems, the order of i and j and M and M' is not relevant. The total spin of the whole system is given by

$$\boldsymbol{S}^2 = (S^x)^2 + (S^y)^2 + (S^z)^2 = \frac{S^+S^- + S^-S^+}{2} + (S^z)^2, \tag{12}$$

where $S^x=\Sigma_i S_i^x$, $S^y=\Sigma_i S_i^y$, and $S^z=\Sigma_i S_i^z$, and $S^\pm=\Sigma_i S_i^x \pm iS_i^y$. We denote the value of the total spin of the system by $S_{\rm tot}$, i.e., $S_{\rm tot}(S_{\rm tot}+1)=\langle \boldsymbol{S}^2\rangle$.

III. CONSERVATION OF THE NUMBER OF PARTICLES OF DIFFERENT SPINS AND GROUND-STATE DEGENERACY

It should be noted that the Hubbard Hamiltonian conserves the number of particles

$$n_M = \sum_{i=1}^{N} n_{i,M} \tag{13}$$

for each M. To describe the set of n_M of states, it is convenient to introduce the notation

$$\{n_M\} = (n_S, n_{S-1}, \cdots, n_{-S}), \tag{14}$$

where $\Sigma_{M=-S}^{+S} n_M=N$. It is important to note that except for $S=1/2$, the operator $S_i^+S_j^-+S_i^-S_j^+$ changes the set $\{n_M\}$. For example, if $S=1$, application of $S_i^+S_j^-+S_i^-S_j^+$ to states in the set $(n_1=0,\ n_0=2,\ n_{-1}=0)$, creates states in the set $(n_1=1,\ n_0=0,\ n_{-1}=1)$. Thus, except for $S=1/2$, the matrix element $\langle\{n_M'\}|S_i^+S_j^-+S_i^-S_j^+|\{n_M\}\rangle$ can be nonzero even if $\{n_M\}\neq\{n_M'\}$. Thus, the operation of $S_i^+S_j^-+S_i^-S_j^+$ on each state with given $\{n_M\}$ must be treated carefully.

Let us denote by $|G,M\rangle$ the ground state in the space with the magnetization M. Because the Hamiltonian $\mathcal{H}$ and $S^\pm$ commute with each other, $S^-|G,M\rangle$ is also a ground state because

$$\mathcal{H}S^-|G,M\rangle = S^-\mathcal{H}|G,M\rangle = E_G S^-|G,M\rangle. \tag{15}$$

However, the set of numbers $\{n_M\}$ is not necessarily conserved, that is,

$$\begin{aligned} S^-\mathcal{H}|G,M,\{n_M\}\rangle &= E_G S^-|G,M,\{n_M\}\rangle \\ &= E_G \sum_{\{n_M'\}} a_{\{n_M'\}}|G,M-1,\{n_M'\}\rangle, \end{aligned} \tag{16}$$

where $|G,M,\{n_M\}\rangle$ denotes one of basis states with fixed $\{n_M\}$ that contribute to the expansion of $|G,M\rangle$ in terms of basis states. The energy of this state is given by

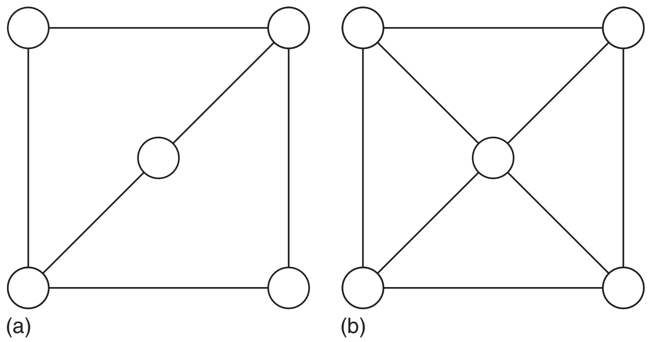

FIG. 1. Lattices: (a) five sites and six bonds; (b) lattice (a) with two additional bonds.

$$E_G = \frac{\langle G,M,\{n_M\}|S^{+}\mathcal{H}S^{-}|G,M,\{n_M\}\rangle}{\langle G,M,\{n_M\}|S^{+}S^{-}|G,M,\{n_M\}\rangle} = \frac{\sum_{\{n'_M\}}|a_{\{n'_M\}}|^2\langle G,M-1,\{n'_M\}|\mathcal{H}|G,M-1,\{n'_M\}\rangle}{\sum_{\{n'_M\}}|a_{\{n'_M\}}|^2\langle G,M-1,\{n'_M\}|G,M-1,\{n'_M\}\rangle}. \tag{17}$$

Because

$$\frac{\langle G,M-1,\{n'_M\}|\mathcal{H}|G,M-1,\{n'_M\}\rangle}{\langle G,M-1,\{n'_M\}|G,M-1,\{n'_M\}\rangle} \geq E_G, \tag{18}$$

in order to satisfy the relation in Eq. (17), the state in each set of numbers $\{n'_M\}$ must be the ground state, i.e.,

$$\frac{\langle G,M-1,\{n'_M\}|\mathcal{H}|G,M-1,\{n'_M\}\rangle}{\langle G,M-1,\{n'_M\}|G,M-1,\{n'_M\}\rangle} = E_G, \tag{19}$$

and thus

$$\mathcal{H}|G,M-1,\{n'_M\}\rangle = E_G|G,M-1,\{n'_M\}\rangle. \tag{20}$$

This shows that the same ground-state energy is found for all sets $\{n'_M\}$ except if $a_{\{n'_M\}}=0$. This structure of degeneracy reflects the fact that the rank of the SU($2S+1$) is larger than 1 for $S>1/2$. If the ground state is a fully symmetric state, its degeneracy is given by the multiplets of the fully symmetric Young tableau of a given number of particles, as we demonstrate in the following sections.

IV. GROUND STATE OF A FERMION SYSTEM WITH $S=\frac{3}{2}$

A. Subspaces due to conservation of particle number of each M

Let us now study the dependence of the ground-state energy on U_0. We illustrate the properties of the model by considering a simple system with four particles. We study the case of $S=3/2$ on the five-site lattice depicted in Fig. 1(a). Qualitatively, our conclusions do not change if we consider more complicated lattices. Figure 2(a) shows the ground-state energies for the sets $\{n_M\}=(n_{3/2},n_{1/2},n_{-1/2},n_{-3/2})=(4,0,0,0),(3,1,0,0),(2,2,0,0),(2,1,1,0),(1,1,1,1)$, corresponding to systems in which the number of different states is 5, 50, 100, 250, and 625, respectively. The energy does not depend on the value of M, and thus the energy-level structure is same for the cases with the same set of numbers, e.g., for $(3,1,0,0),(3,0,1,0),\cdots,(0,0,1,3)$. For a small value of U_0, the ground-state energies are all different. As U_0 increases, the ground-state energies of (3,1,0,0), (2,2,0,0), and (2,1,1,0) become degenerate with that of (4,0,0,0). This fact indicates that the lowest energy state in these sets has total spin $S_{\max}$. However, we find that the energy of the lowest energy state in the set (1,1,1,1) is always smaller than that of (4,0,0,0). This means that even at large U_0 the ground state has total spin $S<S_{\max}$. Therefore, in the present case the Nagaoka-ferromagnetic state, that is, the state of the maximum total spin, is not the ground state, even at large values of U_0.

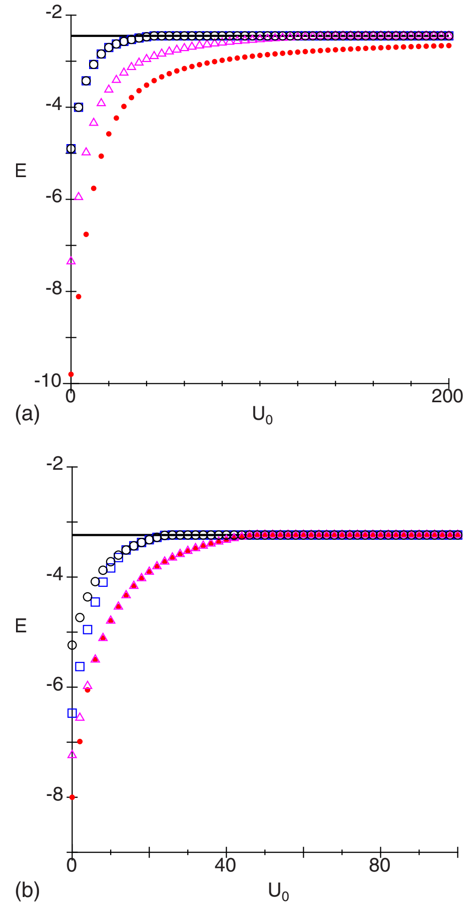

FIG. 2. (Color online) Energies of the ground state for several sets of $(n_{3/2},n_{1/2},n_{-1/2},n_{-3/2})$. (a) A system with four particles on the lattice shown in Fig. 1(a). The solid line denotes the ground-state energy for (4,0,0,0), circles: (3,1,0,0), squares: (2,2,0,0), triangles: (2,1,1,0), and bullets: (1,1,1,1). (b) Same as (a) but for the lattice shown in Fig. 1(b).

In general, when there are multiple conserved quantities that do not commute with each other, each energy eigenstate of the system is usually degenerate as shown by Eq. (20).[15] In the present model, the total magnetization and the set $\{n_M\}$ are conserved. In fact, the model is invariant under the SU(4) operation whose rank is three. The eigenstates belong to various subspaces represented by the Young tableau. In the case where a state with the maximum total spin belongs to the

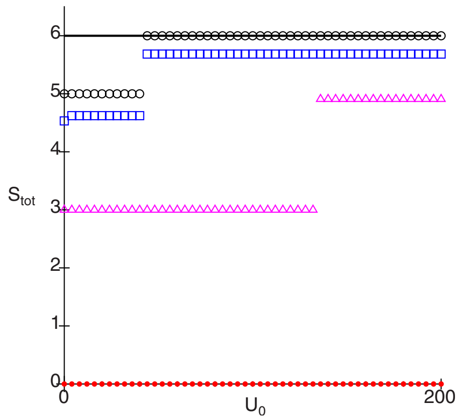

FIG. 3. (Color online) The total spin S_{tot} of the lowest energy state for the cases shown in Fig. 2(a). The legend is the same as in Fig. 2.

ground state, the ground state has the multiplets of the fully symmetric Young tableau. The ground state depicted in Fig. 2(a) is not of this type, but belongs to some other subspace.

Let us study the total spin of the states in Fig. 2(a). Figure 3 shows the U_0 dependence of the total spin of the lowest energy state for the sets $\{n_M\}$ on the lattice of Fig. 2(a). The total spin of the ground state is zero for all U_0, and thus we find that the state is spin singlet.

Next, we also study the change in the total spin of other states as a function U_0. The total spin of states in the subspace (4,0,0,0) is $S_{max}=(3/2)\times 4=6$. For small values of U, the total spin of states in the subspace (3,1,0,0) is less than S_{max}, but it becomes S_{max} when the ground-state energy becomes degenerate to that of the (4,0,0,0) subspace. The ground-state energies of (2,2,0,0) and (2,1,1,0) become degenerate to that of (4,0,0,0) at certain values of U_0. However, the total spin of the lowest energy state of these sets does not reach S_{max}. This fact agrees with the earlier argument that if $S^-|G,M\rangle$ consists of more than one set of $\{n_M\}$, neither of these sets yields an eigenstate of the total spin although each of them have the same ground-state energy. Therefore, the expectation value of the total spin is not necessarily an integer.

B. Connectivity condition for the system with $S=\frac{3}{2}$

Note that the lattice shown in Fig. 1(a) satisfies the so-called connectivity condition for the case of $S=1/2$ and the Nagaoka-ferromagnetic state is realized at large U_0. However, for $S=3/2$, we find that the fully symmetric state is not a ground state. As is well known,[6,7] the Nagaoka-ferromagnetic state is realized when we can produce all possible configurations by hopping without producing doubly occupied sites (the connectivity condition). For $S=1/2$ (two types of particles) and the lattice shown in Fig. 1(a), this condition is satisfied and the Nagaoka-ferromagnetic state is realized. However, in the case of $S=3/2$ particles with different magnetizations, i.e., $M=-3/2,-1/2,1/2,3/2$, the connectivity condition is not satisfied. Indeed, the configuration depicted in Fig. 4(a) cannot be changed into that of Fig. 4(b) by hopping only.

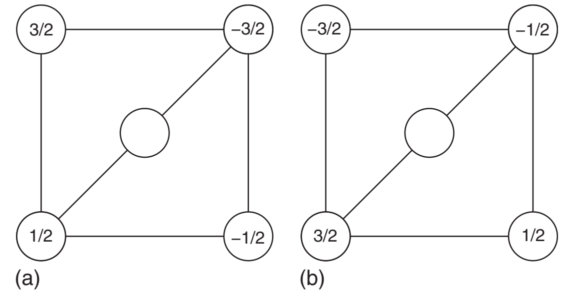

FIG. 4. Nearest-neighbor hopping cannot change configuration (a) into (b) without creating double occupancy.

In order to re-establish the connectivity condition we may add bonds and construct the lattice depicted in Fig. 1(b). This lattice has loops with an odd number of bonds and, therefore, we need to change the sign of some hopping integrals. Then, we find that the Nagaoka-ferromagnetic state is realized, as is clear from Fig. 2(b). Summarizing, we have shown that $S=3/2$ itinerant particle systems may exhibit Nagaoka ferromagnetism although the connectivity condition is more difficult to satisfy.

C. Structure of the ground state

We found that the ground state in each subspace for a set $\{n_M\}$ is unique but in each subspace we have one state with the same energy. In Table I, we list all the sets $\{n_M\}$. The structure indicates that there is a state with $S_{tot}=6$, 4, 2, and 0. In the case of $S=1/2$, i.e., the case SU(2) whose rank is one, there is a one-to-one correspondence between the magnetization M and $\{n_+,n_-\}$, as is clear from Table I. Thus, when we create states by applying S^- from the all-up state, we have only the state with $S_{tot}=2$ for the four-spin system. In contrast, in the case of $S=3/2$, i.e., the case SU(4) whose rank is three, we create different sets by applying S^-. As we mentioned in Sec. III, the created states are degenerate ground states. Therefore, the structure displayed in Table I is intrinsic for systems with $S=3/2,5/2,\ldots$. The number of states is 35, which is the number of multiplets of the fully symmetric Young tableau of length four for SU(4).

V. GROUND STATE OF A BOSON SYSTEM WITH $S=1$

Next, we study a system of $S=1$ particles, namely, a boson system. Figure 5 shows the ground-state energies for the sets $\{n_M\}=(n_1,n_0,n_{-1})=(4,0,0),(0,4,0),(3,1,0),(3,0,1),(2,2,0),(2,1,1),(1,2,1)$, corresponding to systems in which the number of different states is 70, 70, 175, 175, 225, 375, and 375, respectively. Surprisingly, we find that the energies are the same for all the cases, regardless of U_0. The total spin of the different ground states are also plotted in Fig. 5. It is clear that the magnetic properties of the boson system are very different from that of the fermion system. The values of the total spin are less than the maximum value ($S_{tot}=4$) except for (4,0,0) and (3,1,0), which reflects the fact that the eigenstate of maximum total spin contains more than two sets of $\{n_M\}$, as in the case of the $S=3/2$ system.

TABLE I. Sets of particles of different $M(\geq 0)$ $\{n_M\}$ for the cases of $S=3/2$ and 1/2. For $S=3/2$, we list the sets for positive M only.

			$S=3/2$				
M	6	5	4	3	2	1	0
$\{n_M\}$	(4,0,0,0)	(3,1,0,0)	(3,0,1,0)	(3,0,0,1)	(0,4,0,0)	(0,3,1,0)	(1,0,3,0)
			(2,2,0,0)	(1,3,0,0)	(2,1,0,1)	(2,0,1,1)	(0,3,0,1)
				(2,1,1,0)	(1,2,1,0)	(1,2,0,1)	(2,0,0,2)
					(2,0,2,0)	(1,1,2,0)	(0,2,2,0)
							(1,1,1,1)
			$S=1/2$				
M	2	1	0	−1	2		
$\{n_M\}$	(4,0)	(3,1)	(2,2)	(1,3)	(0,4)		

We find a ground state in each set, just as in the case of $S=3/2$. We list the sets in Table II. Again we find degenerate ground states with $S_{\mathrm{tot}}=4$, 2, and 0, which is an intrinsic property of systems with $S>1/2$. The degeneracy of the ground state is 15, which is the number of the multiplets of fully symmetric Young tableau of length four for SU(3).

Moreover, we find that even in the half-filled case, in the boson system the total spin takes the maximum value. For instance, for a system of four atoms on a simple square lattice (corresponding to the half-filled case), the ground state is the same for all sets $\{n_M\}$ (results not shown).

This property of boson systems can be understood as follows. Consider the subspace for a fixed set $\{n_M\}$. All the off-diagonal matrix elements of the Hamiltonian $\mathcal{H}$ are $-t$ or zero. By subtracting an appropriate multiple of the unit matrix, also the diagonal elements can be made negative or zero. Thus, all the elements of the shifted matrix $\widetilde{H}$ are either negative or zero. In our model, all the sites are connected by bonds and, therefore, there exist a number $n>0$ such that all the elements of $\widetilde{H}^{2n}$ are positive. Then, the Perron-Frobenius theorem tells us that there exist a unique eigenstate of $\widetilde{H}^{2n}$ with an eigenvalue that is larger than the absolute value of all other eigenvalues. This unique eigenstate is, therefore, the ground state of H, which is totally symmetric with positive coefficients, and contains the state of the maximum total spin. However, as we mentioned above, this ground state has an intrinsic degeneracy with respect to subspaces that have different $\{n_M\}$ because we can always start from the state with all maximum spins and let S^- create different sets of $\{n'_M\}$. The states that are generated in this manner have all positive coefficients and are, therefore, ground states too. Clearly, this property does not depend on the connectivity of the lattice or on the value of U_0.

In contrast, for fermion systems, it is in general impossible to transform H such that all elements have the same sign but in those cases for which such a transformation exist, which is precisely the condition of Nagaoka ferromagnetism, we can apply the same arguments as in the boson case to prove that the ground state of H is totally symmetric with positive coefficients and contains the state of the maximum total spin.

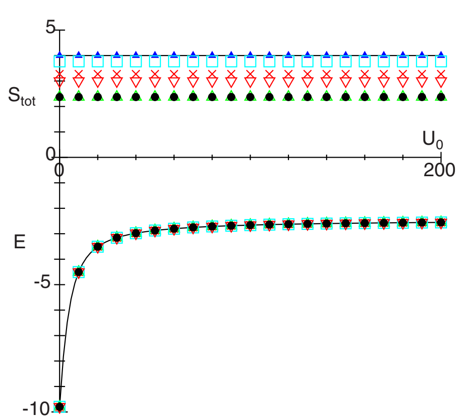

FIG. 5. (Color online) Ground-state energies E and the corresponding total spin S_{tot} as a function of U_0 for four $S=1$ bosons and various (n_1,n_0,n_{-1}) on the lattice shown in Fig. 1(a). The solid line denotes data for (4,0,0), solid triangles: (3,1,0), open triangles: (3,0,1), open squares: (2,2,0), reversed triangles: (2,1,1), crosses: (1,2,1), and bullets: (0,4,0).

VI. SUMMARY AND DISCUSSION

We have studied the magnetic properties of the ground state of itinerant systems with $S>1$. We found that fermion systems ($S=3/2$) support an extended form of Nagaoka ferromagnetism but that the connectivity condition is more difficult to satisfy because of the presence of particles with different magnetizations. When the maximum S_{tot} state is the ground state, the system has a degenerate ground state in each set $\{n_M\}$ listed in Table I. Thus the ground-state mani-

TABLE II. Sets of particles of different $M(\geq 0)$ $\{n_M\}$ for the case of $S=1$.

			$S=1$		
M	4	3	2	1	0
$\{n_M\}$	(4,0,0)	(3,1,0)	(3,0,1)	(2,1,1)	(0,4,0)
			(2,2,0)	(1,3,0)	(2,0,2)
					(1,2,1)

fold consists of not only the state of the maximum $S_{\rm tot}$ but also of states with smaller values of $S_{\rm tot}$, reflecting the SU(4) symmetry of the model. This degenerate structure is intrinsic for the systems with $S>1/2$, where several sets of $\{n_M\}$ exist for a given value of M.

For boson systems ($S=1$) we found that there is a ground state in each set of $\{n_M\}$ and that, due to the SU(3) symmetry, the same type of degenerate ground-state structure appears as the one found for $S=3/2$, but in this case regardless of the value of U_0 and of the shape of the lattice, which is an intrinsic property of the bosonic case. Thus, we conclude that boson itinerant magnetic systems always have a state with the maximum total spin belonging to the manifold of ground states. This property follows quite naturally from the fact that the Hamiltonian of the boson system can be transformed such that all elements have the same sign, implying that the ground state is fully symmetrized. In contrast, for fermion systems we need an additional condition, the condition for the Nagaoka ferromagnetism, for the ground state to have the maximum total spin. We expect that these properties will be confirmed in real experimental systems.

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[1] W. Heisenberg, Z. Phys. **38**, 411 (1926); **49**, 619 (1928).
[2] M. Roger, J. H. Hetherrington, and J. M. Delriew, Rev. Mod. Phys. **55**, 1 (1983).
[3] T. Momoi, P. Sindzingre, and N. Shannon, Phys. Rev. Lett. **97**, 257204 (2006).
[4] J. Hubbard, Proc. R. Soc. London, Ser. A **276**, 238 (1963).
[5] Y. Nagaoka, Phys. Rev. **147**, 392 (1966).
[6] A. Mielke and H. Tasaki, Commun. Math. Phys. **158**, 341 (1993).
[7] H. Tasaki, Prog. Theor. Phys. **99**, 489 (1998).
[8] E. Müller-Hartmann, J. Low Temp. Phys. **99**, 349 (1995).
[9] W. O. Putikka, M. U. Luchini, and M. Ogata, Phys. Rev. Lett. **69**, 2288 (1992).
[10] Y. Watanabe and S. Miyashita, J. Phys. Soc. Jpn. **66**, 2123 (1997); **68**, 3087 (1999).
[11] S. Miyashita, Prog. Theor. Phys. **120**, 785 (2008).
[12] M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature **415**, 39 (2002); L. Santos, M. A. Baranov, J. I. Cirac, H.-U. Everts, H. Fehrmann, and M. Lewenstein, Phys. Rev. Lett. **93**, 030601 (2004).
[13] Y. Takahashi (private communication).
[14] C. Honerkamp and W. Hofstetter, Phys. Rev. Lett. **92**, 170403 (2004); A. Rapp, W. Hofstetter, and G. Zarand, Phys. Rev. B **77**, 144520 (2008); A. V. Gorshkov, M. Hermele, V. Gurarie, C. Xu, P. S. Julienne, J. Ye, P. Zoller, E. Demler, M. D. Kukin, and A. M. Roy, arXiv:0905.2610 (unpublished); M. A. Cazalilla, A. F. Ho, and M. Ueda, arXiv:0905.4948 (unpublished).
[15] L. D. Landau and E. M. Lifshitz, *"Quantum Mechanics" Course of Theoretical Physics* (Elsevier, Amsterdam, 1977).